MULTIVALUED DECISION DIAGRAMS: ALGORITHMS AND APPLICATIONS

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Abstract

Multivalued decision Diagram are more and more used in optimization. We aim at showing that we can expect to directly use MDDs to build complex models. We have defined two reduction operations, two combination operators, several building methods and schemes. Thanks to them, we have successfully solved real world hard problems in music and data processing.

Reduction

Reduction

- ❑ Operation which **merge *equivalent* nodes**
- ❑ Two nodes ***x*** and ***x'*** are equivalent ***iff*** :

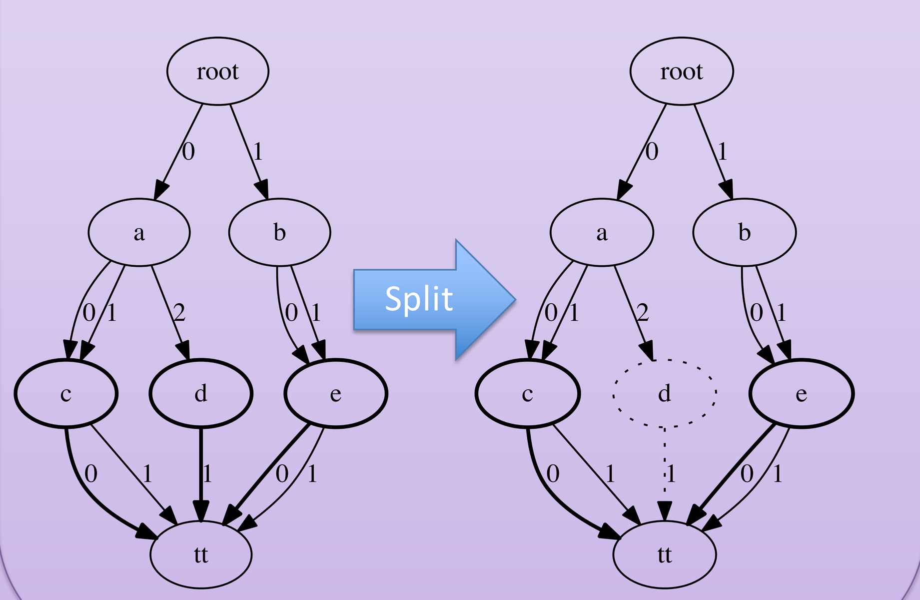
$\forall (\boldsymbol{x}, v, y) \in \boldsymbol{\omega}^{+}(\boldsymbol{x}),\ \exists (\boldsymbol{x'}, v, y) \in \boldsymbol{\omega}^{+}(\boldsymbol{x'})$

$|\boldsymbol{\omega}^{+}(\boldsymbol{x})| = |\boldsymbol{\omega}^{+}(\boldsymbol{x'})|$

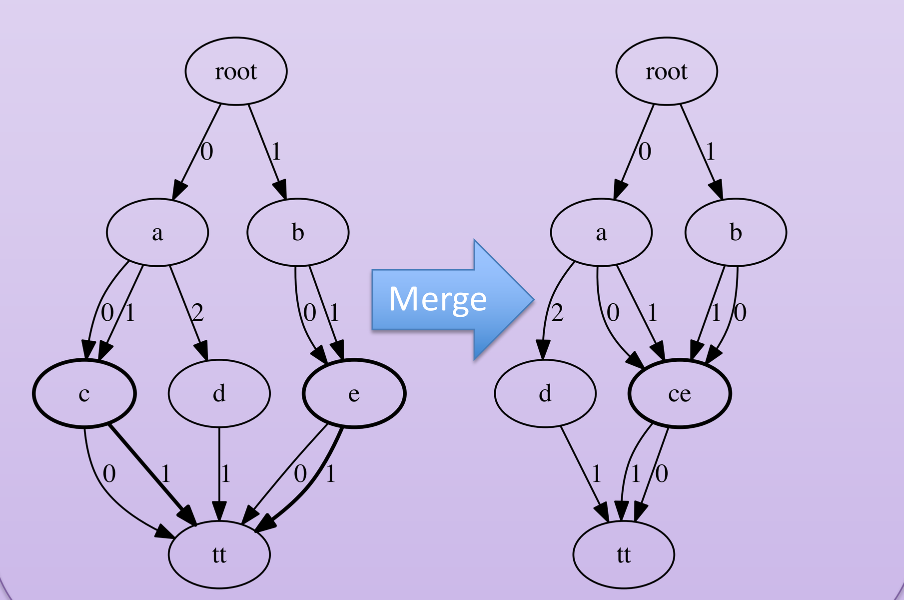
Two methods

- ❑ **Both linear**
- ❑ **Incremental**
  - ❑ **ipReduce**
- ❑ **Or not**
  - ❑ **pReduce**

Compare and split the Pack



Merge equivalent nodes



Creations

Regular creation

- ❑ Based on an automaton
- ❑ Cannot be larger

Dynamic programming

- ❑ Knapsack
- ❑ Independent set
- ❑ AllDifferent

Extensional Data

- ❑ Table
- ❑ GCS – Tuple sequences
- ❑ Extraction of solutions

Pattern creation

- ❑ No global function

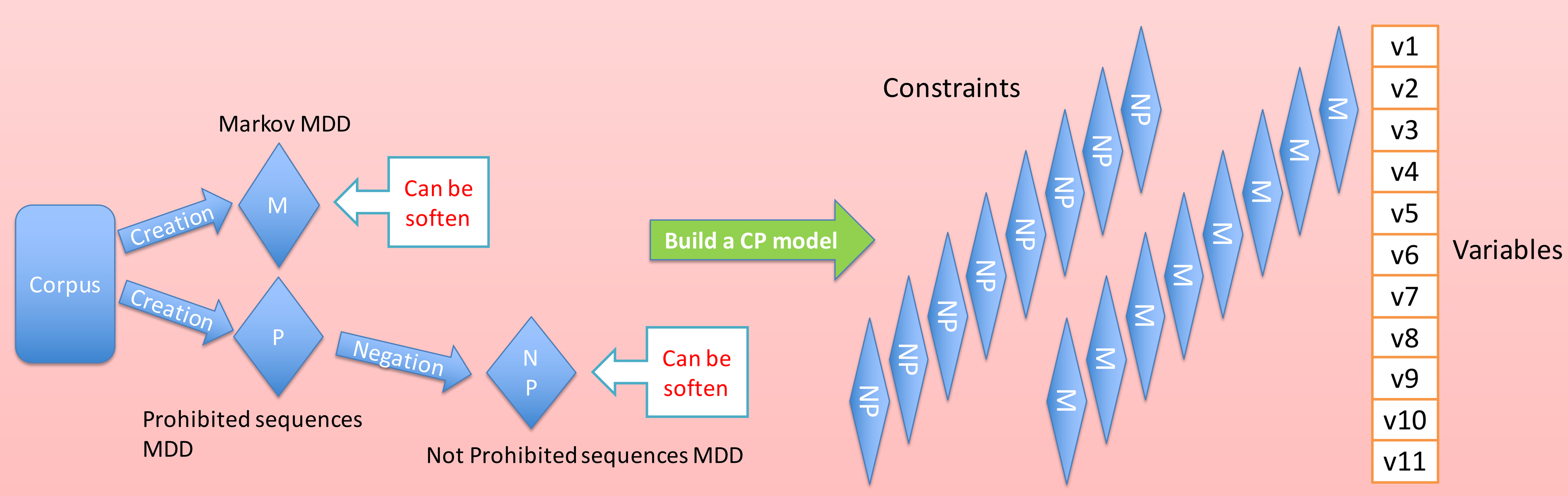
Advantages

Multivalued Decision Diagrams allow the **compression** of the data it contains. An MDD is **always smaller** than the linear representation. Furthermore, all the construction provided allow to **build MDD** representing a huge amount of tuples **without their enumeration**.

Applications

MaxOrder Problem

Generate sequences of words from a corpus where all the subsequences of length 2 belong to the corpus and none subsequences of length 4 belong to the corpus.



Allen Constraint using MDDs & Enforcing Structure on Temporal Sequences

The Allen constraint using MDD allows to constrain part of a temporal sequences with respect to temporal position and not the index of the variables.

Applications:

Audio multi-track synchronization

Lead-Sheet generation

MDD & Constraints

Goal: Allow solvers to deal with MDDs

MDD4R constraint

- ❑ MDD constraint
- ❑ Linear amortized

Cost-MDD4R constraint

- ❑ Edge with cost
- ❑ Bound the paths

Soft constraint

- ❑ Allow the use of a certain amount of invalid edges

Results

MaxOrder Problem Solve time

Size	MDD model	State of the art
8	6 s	50 minutes
20	26.8 s	Memory out

MaxOrder Build Time: Only our method scale

Audio multi-track synchronization

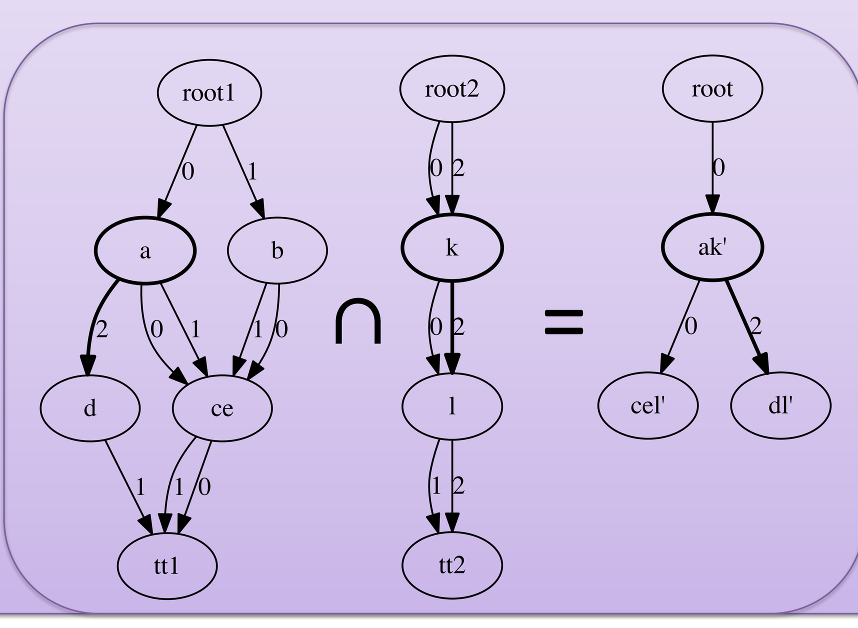
Size	MDD model	State of the art
2	< 1 second	5.4 seconds
3	< 1 second	Time-Out
14	112 seconds	Time-Out

Combination

Creation of an edge (**x**, v, y) **function** of

- ❑ $(\boldsymbol{x_1}, v, y_1) \in \boldsymbol{\omega}^{+}(\boldsymbol{x_1})$
- ❑ $(\boldsymbol{x_2}, v, y_2) \in \boldsymbol{\omega}^{+}(\boldsymbol{x_2})$

4 possible states



Generic operator

MDD$_r$ is a **function** of the two MDDs

Each node **x** $\in$ mdd$_r$ is a combination of:

- ❑ $\boldsymbol{x_1} \in$ mdd$_1$
- ❑ $\boldsymbol{x_2} \in$ mdd$_2$

Need to distinguish the **level**

- ❑ **Level** $\in$ 1 … r-1
- ❑ **Level** = r

Possible operations:

- Intersection
- Union
- Difference
- …

In-place operations:

- Addition & deletion